

Series of scientific works

Quantum Effects of Interaction in Nanosystems

Gleb A. Skorobagatko

Institute for Condensed Matter Physics of National Academy of Sciences of Ukraine

Lviv

2019

KEY FEATURES OF SINGLE-ELECTRON TRANSPORT IN QUANTUM DOTS

Basic theoretical idea on sigle-electron charge transport in nano-sized granules: "A quantitative theory in terms of rate equations describing the transport through a blockaded quantum dot or metal grain at $G \ll e^2/2\pi\hbar$ was formulated in Refs. [1,2], and was generalized to systems with a controllable gate in Ref.[3]." – from the Review article by: I.L. Aleiner, P.W. Brouwer, L.I. Glazman in: Physics Reports, V. 358, No.5-6, 2002, Pp. 309-440.



Example of experimental design of nanotube-based single-electron transistor:

Typical values of main characteristic parameters for single-electron transistors, as relevant nanosystems :

Characteristic energies and differences between chemical potentials of the leads (bias voltages) in such systems: < 100 meV

Characteristic temperatures: < 100 mK

Characteristic scale of a system: 10-100 nm



1. INTERPLAY OF QUANTUM-VIBRATIONAL AND LUTTINGER LIQUID EFFECTS IN ELECTRON TRANSPORT THROUGH SINGLE-LEVEL QUANTUM DOT. Model.

G.A.Skorobagatko, I.V.Krive, Low Temp. Phys. 34, 858 (2008).

Model of a single-electron molecular transistor with Luttinger liquid leads and quantum vibrating quantum dot:



Total Hamiltonian of the system:

$$\mathcal{H} = \mathcal{H}_{LL} + \mathcal{H}_{QD} + \mathcal{H}_T$$

 $\mathcal{H}_{LL} = \sum_{j=L,R} \mathcal{H}_l^{(j)}$

- Hamiltonian of Luttinger liquid leads in bosonic representation («charge» sector):

$$\mathcal{H}_l^{L(R)} = \mathcal{H}_l = \hbar v_c \int_0^\infty a_k^+ a_k k dk$$

Hamiltonian of quantum vibrating quantum dot with electron-vibron interaction:

$$\mathcal{H}_{QD} = \varepsilon_0 f^+ f \notin \varepsilon_i (b^+ + b) f^+ f + \hbar w_0 b^+ b$$

Tunnel Hamiltonian:

$$\mathcal{H}_T = \sum_{j=L,R} \{ t_j f^+ \Psi(j) + h.c. \}$$

$$\Psi(L(R)) = \sqrt{\frac{2}{\pi\alpha}} \cdot \exp\left[\int_0^\infty dk \frac{e^{-\alpha k/2}}{\sqrt{2K_\rho k}} \cdot (a_k - a_k^+)\right]$$

Connection between fermionic and bosonic operators in the leads

Luttinger liquid correlation $g = (1 + U/2\pi v_F)^{-1/2}$ parameter: $K_{\rho} = (2/g - 1)^{-1}$

1. INTERPLAY OF QUANTUM-VIBRATIONAL AND LUTTINGER LIQUID EFFECTS IN ELECTRON TRANSPORT THROUGH SINGLE-LEVEL QUANTUM DOT. Results.

G.A.Skorobagatko, I.V.Krive, Low Temp. Phys. 34, 858 (2008).

Differential conductance: G(V) = dI/dV in the non-linear regime on bias voltage



2. POLARONIC EFFECTS IN THE EMERGENT SHUTTLE INSTABILITY OF QUANTUM DOT CLASSICAL MOTION. Model.

G.A.Skorobagatko, I.V.Krive, R.I.Shekhter, Low Temp. Phys. 35, 949 (2009).

Model of single-electron transistor with vibrational degree of freedom:

(see e.g. <u>L.Y.Gorelik, R.I.Shekhter et al.,</u> <u>Phys.Rev.Lett. -1998. –V.80, No. 20. -P. 4526.</u>)



Total Hamiltonian of the model:

$$\hat{H} = \sum_{j=L,R} \hat{H}_l^{(j)} + \hat{H}_d + \sum_{j=L,R} \hat{H}_t^{(j)}$$

Hamiltonian of Fermi-liquid lead :

$$\hat{H}_l^{(j)} = \sum_k (\varepsilon_k - \mu_j) \hat{a}_{kj}^+ \hat{a}_{kj}$$

Hamiltonian of quantum dot with electronvibron interaction:

$$\hat{H}_d = \varepsilon_0 \hat{c}^+ \hat{c} - \varepsilon_i (\hat{b}^+ + \hat{b}) \hat{c}^+ \hat{c} + \hbar \omega_0 \hat{b}^+ \hat{b}$$

Tunnel Hamiltonian:

$$\hat{H}_{t}^{(j)} = \sum_{k} \left[\hat{t}_{j}(\hat{X}_{c.m.}) \hat{a}_{kj}^{+} \hat{c} + h.c. \right]$$

$$\hat{t}_j(\hat{X}) = t_{0j} \exp(j\lambda_t \hat{X})$$

 $\equiv x_0 / l_t \qquad \qquad j = (L, R) \equiv (-, +)$

2. POLARONIC EFFECTS IN THE EMERGENT SHUTTLE INSTABILITY OF QUANTUM DOT CLASSICAL MOTION. Quantum equations of motion.

G.A.Skorobagatko, I.V.Krive, R.I.Shekhter, Low Temp. Phys. 35, 949 (2009).

Applying first-principles quantum equation of motion (QEOM) method in the regime of <u>electron sequential tunneling</u>: $T \gg \Gamma_{\pm}$ $\Gamma = \Gamma_{0L} + \Gamma_{0R}$ $\Gamma_{0i} = 2\pi \rho_i |\bar{t}_{0i}|^2$

- One can obtain following **«classical» equation of motion** for the average coordinate of molecular shuttle center-of-mass displacement, with the **averaged «quantum» force** y in the r.h.s. of equation:

$$\frac{d^2}{dt^2} \left[\bar{x}(t) + \lambda N\{\bar{x}(t)\} \right] + \omega_0^2 \bar{x}(t) = -\frac{\omega_0 \lambda_t}{\hbar} \sum_{\substack{j = \frac{-/L}{+/R}}} j H_j\{\bar{x}(t)\} - \frac{\mathbf{S}}{\mathbf{N}}$$

$$\frac{\mathbf{N}}{\mathbf{N}} \frac{\mathbf{N}}{\mathbf{N}} \frac{1}{\mathbf{N}} \left\{ \bar{x}(t) \right\} = \langle \hat{c}^+(t)\hat{c}(t) \rangle \qquad H_j\{\bar{x}(t)\} = \langle \hat{H}_j(\hat{X}, \hat{p}) \rangle$$

- See similar calculation in: D.Fedorets , Phys.Rev. B. V. 68, No. 3,P. 033106 (2003)

Linearizing above equation with respect to small parameters:

 $\bar{x}(t) = A_0 e^{r_s t} \sin(t)$

 $\lambda \bar{x}(t) \ll 1 \qquad \lambda_t \bar{x}(t) \ll 1$

searching its solution (in the form:

one obtains following shuttle instability increment:

$$r_s = \sum_{\substack{j = \frac{-/L}{+/R}}} \frac{\bar{\Gamma}_{0j}}{8} \sum_{l=-\infty}^{+\infty} F_{lj}(\beta) \left\{ (\lambda - j\lambda_t)^2 f_j(\Delta - l + 1) - (\lambda + j\lambda_t)^2 f_j(\Delta - l - 1) \right\}$$
$$\bar{\Gamma}_{0j} = \Gamma_{0j} \exp(j\lambda\lambda_t)$$

Where:

$$F_{lj}(\beta) = \exp\left[-(\lambda^2 - \lambda_t^2)(1 + 2n_b)\right] \left|\frac{\lambda + j\lambda_t}{\lambda - j\lambda_t}\right|^l I_l(2|\lambda^2 - \lambda_t^2|\sqrt{n_b(1 + n_b)})e^{-\beta l/2}$$

2. POLARONIC EFFECTS IN THE EMERGENT SHUTTLE INSTABILITY OF QUANTUM DOT CLASSICAL MOTION. Results.

G.A.Skorobagatko, I.V.Krive, R.I.Shekhter, Low Temp. Phys. 35, 949 (2009).



3. ROLE OF MAGNETIC PHASE QUANTUM FLUCTUATIONS IN SEQUENTIAL ELECTRON TUNNELING THROUGH VIBRATING QUANTUM DOT. Model.

G.A. Skorobagatko, S.I. Kulinich, I.V. Krive, R.I. Shekhter, M. Jonson, Low Temp. Phys., 35, (2011).

Model of single-electron molecular transistor with quantum vibrational degree of freedom in the transverse magnetic field



Total Hamiltonian of the model:

$$\hat{H} = \sum_{j=L,R} (H_j^{(l)} + H_j^{(t)}) + H_D$$

Hamiltonian of Fermi-liquid lead:

$$H_j^{(l)} = \sum_k (\varepsilon_{k,j} - \mu_j) a_{k,j}^{\dagger} a_{k,j}$$

Hamiltonian of vibrating quantum dot :

$$H_D = \varepsilon_0 c^{\dagger} c + \hbar \omega_0 b^{\dagger} b$$

Tunnel Hamiltonian:

$$H_j^{(t)} = \underbrace{t_j(\hat{y}) \exp(-ij\hat{\phi}) a_{k,j} c^{\dagger} + H.c.}_{j = (L,R) \equiv (-,+)}$$

$$\hat{\phi} = eHd\hat{y}/\hbar c$$

-Aharonov-Bohm magnetic phase of tunneling electron, in the case of quantum fluctuations of quantum dot coordinate along y-axis

3. ROLE OF MAGNETIC PHASE QUANTUM FLUCTUATIONS IN SEQUENTIAL ELECTRON TUNNELING THROUGH VIBRATING QUANTUM DOT. Results.

G.A. Skorobagatko, S.I. Kulinich, I.V. Krive, R.I. Shekhter, M. Jonson, Low Temp. Phys., 35, (2011).

<u>Results:</u> magneto-polaronic blockade; anomalous temperature dependence of the conductance; "excess" (inelastic) part of average current.



4. ANDREEV-TYPE RESONANT POLARONIC TUNNELING OF STRONGLY INTERACTING ELECTRONS THROUGH VIBRATING QUANTUM DOT. Model.

<u>G.A.Skorobagatko</u> Phys. Rev. B 85, 075310 (2012).

Total Hamiltonian of the system:

$$\hat{H} = \sum_{j=L,R} \hat{H}_l^{(j)} + \hat{H}_d + \sum_{j=L,R} \hat{H}_t^{(j)}$$

where: $\hbar v_g = \hbar v_F/g = 1$

Tunnel Hamiltonian:

$$\hat{H}_t^{(j)} = (\gamma_j \hat{d}^+ \hat{\Psi}_j(0) + h.c.)$$

where:
$$\hat{\Psi}_j(x) = \exp(i\Phi_j(x)/\sqrt{g})/\sqrt{2\pi a_0}$$

 $\hat{\Psi}_j(0) = (\hat{\Psi}_j(0^-) + \hat{\Psi}_j(0^+))/2$

Chiral charge-density bosonic operator in the leads:

$$\hat{\rho}_j(x) = \hat{\Psi}_j^+(x)\hat{\Psi}_j(x)$$
$$= \partial_x \Phi_j(x)/2\pi\sqrt{g}.$$

Hamiltonian of Luttinger liquid lead with interacting electrons in its bosonic representation:

$$\hat{H}_l^{(j)} = 1/4\pi \int dx (\partial_x \Phi_j(x))^2 \quad (j = L, R)$$

Hamiltonian of quantum dot with electron-vibron- and fixed Coulomb interactions:

$$\begin{aligned} \hat{H}_{d} &= \{ \Delta \hat{d}^{+} \hat{d} + \lambda_{C} \hat{d}^{+} \hat{d} \sum_{j=L,R} \hat{\Psi}_{j}^{+}(0) \hat{\Psi}_{j}(0) \} + \\ &+ \hbar \omega_{0} \{ \frac{1}{2} (\hat{p}^{2} + \hat{x}^{2}) + \lambda \hat{x} \hat{d}^{+} \hat{d} \} \end{aligned}$$

Bosonization and re-fermionization:

$$\begin{aligned}
\hat{\Psi}_{\pm}(x) &= (\Phi_L(x) \pm \Phi_R(x))/2, \\
\hat{U}_f &= \exp[-i(\hat{d}^+\hat{d} - 1/2)\Phi_+(0)/\sqrt{2g}] & \hat{U}_b &= \exp(-i\lambda\hat{p}\hat{d}^+\hat{d}) \\
\\
\hline
\\
Transformed tunnel Hamiltonian: & g = 1/2 & \text{i} \lambda_C = 2\pi \\
\hat{H}_t &= \hat{d}^+\hat{X}^+[\gamma_L\hat{\Psi}_-(0) + \gamma_R\hat{\Psi}_-^+(0)] + \hat{\Psi}_{\pm}(x) &= \exp(i\Phi_{\pm}(x))/\sqrt{2\pi a_0} \\
&+ [\gamma_L\hat{\Psi}_-^+(0) + \gamma_R\hat{\Psi}_-(0)]\hat{X}\hat{d}. & \hat{X} &= \exp(i\lambda\hat{p})
\end{aligned}$$

4. ANDREEV-TYPE RESONANT POLARONIC TUNNELING OF STRONGLY INTERACTING ELECTRONS THROUGH VIBRATING QUANTUM DOT . Results.

<u>G.A.Skorobagatko</u> Phys. Rev. B 85, 075310 (2012).



Effective transmission coefficient in the model, as function of energy of incident electron:



5. Magnetopolaronic Majorana-resonant level (M-MRL-) model G.Skorobagatko, Cond.Mat.Phys., 21(2), 23703, (2018)

Schematic picture of tunnel junction with vibrating CNT as quantum dot in the magnetopolaronic g=1/2 - case:



5. Results of the paper: G.Skorobagatko, Cond.Mat.Phys., 21(2), 23703, (2018): Both constructive and destructive interference between different virtual vibronic channels of resonant magnetopolaronic tunneling – manifestation of the anomalous magneto-polaronic (or "Aharonov-Bohm") blockade effect



6-7. Measurement-caused decoherence and detection of a qubit state: different realizations



Concept of charge qubit: decoherence of the elecron state in Josephson junction due to interaction with FL QPC (Aleiner, Wingreen, Meir, PRL, 1997)



Well-known examples

-Known for the case of "free" fermions only (!) -QPCs with Fermi liquid leads(!)

However "in reality" electron-electron interaction in QPC matters (!)

General concept of a QPC as a quantum limited detector (Averin, Sukhorukov PRL,2005)



6-7. MODEL HAMILTONIAN

in papers: G.Skorobagatko, A.Bruch, S.V.Kusminskiy, A.Romito, Phys.Rev. B, 95, 205402, (2017) and G.Skorobagatko, Phys.Rev.B, 98,045409 (2018)

$$\begin{split} H_{\Sigma} &= H_{LL} + H_{QD} + H_{int} \\ H_{LL} &= \frac{1}{2\pi} \sum_{j=L,R} v_g \int_{-\infty}^{0} \left\{ g(\partial_x \varphi_j)^2 + \frac{1}{g} (\partial_x \theta_j)^2 \right\} dx & \text{Hamiltonian of two semi-infinite Luttinger liquids} \\ H_{QD} &= \sum_{n=1,2} \varepsilon_n c_n^{\dagger} c_n + \int_{-\infty}^{0} \left(c_1^{\dagger} c_2 + c_2^{\dagger} c_1 \right) & \text{Charge-qubit part} \\ H_{int} &= \sum_{n=1,2} [\lambda_n \partial_x \theta_+ + \tilde{\lambda}_n \cos (\varphi_- + \text{eVt})]|_{x=0} c_n^{\dagger} c_n & \text{Iunnel Hamiltonian in the "weak link" approximation (Kane, Fisher, PRB, 1992)} \\ \theta_{\pm} &= [\theta_L \pm \theta_R] \\ \varphi_{\pm} &= [\varphi_L \pm \varphi_R] & [\theta_{\alpha}(x), \varphi_{\alpha'}(x')] = 2i\pi \text{sgn}(x - x') \delta_{\alpha, \alpha'} \\ \text{Non-local bosonic phase fields} & \alpha = \pm \\ \text{(important ingredient)} \end{split}$$

6. Results from the paper: G.Skorobagatko, A.Bruch, S.V.Kusminskiy, A.Romito, Phys.Rev. B, 95, 205402, (2017).



6. Results from the paper: G.Skorobagatko, A.Bruch, S.V.Kusminskiy, A.Romito, Phys.Rev. B, 95, 205402, (2017).



FIG. 3. (Color Online) Log-plot of the ratio between local density- and tunneling-induced decoherence as a function of (a) bias voltage and (b) temperature, cf. Eqs. (11), (14). For increasing interaction strength g = 1; 0.9; 0.6; 0.5; 0.3 from light to dark red and from continuous to coarsely dashed. $\frac{T}{\Lambda_g} = 0.01$ in (a) and $\frac{eV}{\Lambda_g} = 0.01$ in (b), $\lambda_2 - \lambda_1 = \lambda_2 - \lambda_1 = \Delta \lambda$.



FIG. 5. (Color Online) Total detector efficiency Q for the interacting case, as function of the applied bias (a) and temperature (b). Different curves from light to dark red and from continuous to coarsely dashed are for increasing interaction strengths, g = 1; 0.9; 0.6; 0.5; 0.3. The figures show strong dependence of Q on g once orthogonality effects are considered. The inset in (b) shows a zoom in of the regime of cross over between monotonous and non-monotonous temperature dependence for $g \sim 0.5$ (g = 0.55; 0.525; 0.5; 0.475; 0.45 from light to dark blue and from continuous to coarsely dashed). We used $\eta = 0.5$, and $\frac{T}{\Lambda_{\pi}} = 0.01$ in (a), $\frac{aV}{\Lambda_{\pi}} = 0.01$ in (b).



7. Results from the paper: G.Skorobagatko, Phys.Rev.B, 98,045409 (2018)

New "Kondo-like" regime of quantum detection for Luttinger liquid quantum detector at temperatures near absolute zero

S-Theorem gives precise answer: S_a = S_d = 0 (!)

Hence, results: $f_1(T) \cong w_1(T)$

$$\Gamma(\Delta \tilde{\lambda}, T) = \left(\frac{\Delta \tilde{\lambda}}{\Lambda_g}\right)^2 f_1(T)$$
$$W(\Delta \tilde{\lambda}, T) = \left(\frac{\Delta \tilde{\lambda}}{\Lambda_g}\right)^2 w_1(T)$$

$$Q_{K} = \frac{1}{1 + \left(\frac{\Lambda_{g}}{\Delta \tilde{\lambda}}\right)^{2} \frac{\tilde{\Gamma}_{K}(\Delta \lambda)}{f_{1}(T)}}$$

and, as well, all the "Kondo-like" description near T=0:

$$\widetilde{\Gamma}_{\kappa}(\Delta\lambda) \approx \frac{\Lambda_{g}}{\pi} \exp\left[-\frac{1}{g}\left(\frac{\Lambda_{g}}{\Delta\lambda}\right)^{2}\right]$$
- ARE EXACT (!)

Remarkably, this limit implies
(see the plot): $\frac{1}{\sqrt{2}} \le \alpha \le 1$
- ARE EXACT (!)

7. It turns out, that theory of low-temperature instability of quantum detection from G.Skorobagatko, Phys.Rev.B, 98,045409 (2018) can explain unusually long timescale for Rabi oscillations damping in modern experiments with charge-qubits

Charge-qubit operation of an isolated double quantum dot

J. Gorman,¹ D. G. Hasko,¹ and D. A. Williams² ¹Microelectronics Research Centre, University of Cambridge, Cambridge CB3 0HE, United Kingdom ²Hitachi Cambridge Laboratory, Hitachi Europe Ltd., Cambridge CB3 0HE, United Kingdom

(a) R Measurement Manipulation (b) $\Psi = a|0\rangle + b|1\rangle$ $\Psi = |0\rangle = |L\rangle$ 0) $|1\rangle$ $V_{\rm p}$ time Δt $V_{\rm p} = 0$ Τ, 0

Experimental effect explanation in G.Skorobagatko, Phys.Rev.B, 98,045409 (2018)







SET